ME 220 – Example of Roll, Pitch, Yaw Errors

Athulan Vijayaraghavan

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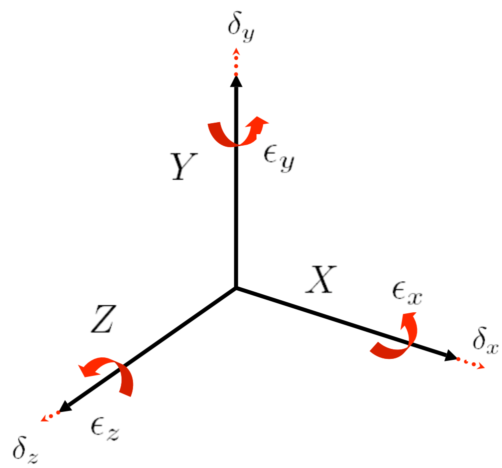

Figure 1: System A with positional errors in $\delta_x, \delta_y, \delta_z$ and rotational errors in $\epsilon_x, \epsilon_y, \epsilon_z$

Consider the coordinate system shown in Figure 1. The rotational system is defined as follows:

- Rotation in **x** is **roll**
- Rotation in **y** is **yaw**
- Rotation in **z** is **pitch**

A *roll* of ϵ_x leads to:

$$\sin\epsilon_x \text{ in } +Z \text{ direction for } Y \Rightarrow O_{ky} \approx \epsilon_x$$
$$1 - \cos\epsilon_x \text{ in } +Y \text{ direction for } Y \Rightarrow O_{jy} \approx 1$$
$$-\sin\epsilon_x \text{ in } +Y \text{ direction for } Z \Rightarrow O_{jz} \approx -\epsilon_x$$
$$1 - \cos\epsilon_x \text{ in } +Z \text{ direction for } Z \Rightarrow O_{kz} \approx 1$$

A *yaw* of ϵ_y leads to:

$$\sin\epsilon_y \text{ in } +Z \text{ direction for } X \Rightarrow O_{kx} \approx \epsilon_y$$
$$1 - \cos\epsilon_y \text{ in } +X \text{ direction for } X \Rightarrow O_{ix} \approx 1$$
$$-\sin\epsilon_y \text{ in } +X \text{ direction for } Z \Rightarrow O_{iz} \approx -\epsilon_y$$
$$1 - \cos\epsilon_y \text{ in } +Z \text{ direction for } Z \Rightarrow O_{kz} \approx 1$$

A *pitch* of ϵ_z leads to:

$$\sin\epsilon_x \text{ in } +Y \text{ direction for } X \Rightarrow O_{jx} \approx \epsilon_z$$
$$1 - \cos\epsilon_x \text{ in } +X \text{ direction for } X \Rightarrow O_{ix} \approx 1$$

$$-\sin\epsilon_x \text{ in } +X \text{ direction for } Y \Rightarrow O_{iy} \approx -\epsilon_z$$
$$1 \text{ - } \cos\epsilon_x \text{ in } +Y \text{ direction for } Y \Rightarrow O_{jy} \approx 1$$

For a case of a system A with roll, pitch, and yaw errors of $\epsilon_x, \epsilon_z, \epsilon_y$ respectively, the HTM with respect to the reference R is:

$$^{R}T_A = \begin{pmatrix} 1 & -\epsilon_z & \epsilon_y & 0 \\ \epsilon_z & 1 & -\epsilon_x & 0 \\ -\epsilon_y & \epsilon_x & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

If this system also had positional errors of $\delta_x, \delta_y, \delta_z$ in the X, Y, Z directions, then the HTM becomes:

$$^{R}T_A = \begin{pmatrix} 1 & -\epsilon_z & \epsilon_y & \delta_x \\ \epsilon_z & 1 & -\epsilon_x & \delta_y \\ -\epsilon_y & \epsilon_x & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

Finally, consider that this system also had the deterministic displacement of x, y, z in each of the principal directions, then the HTM becomes:

$$^{R}T_A = \begin{pmatrix} 1 & -\epsilon_z & \epsilon_y & x + \delta_x \\ \epsilon_z & 1 & -\epsilon_x & y + \delta_y \\ -\epsilon_y & \epsilon_x & 1 & z + \delta_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

ME 220 Fall 2008
Error Budget Examples

1 HTM of a Component with Rotational and Displacement Errors

In this example we develop the HTM for a component with rotational and displacement errors (please see Figure 1).

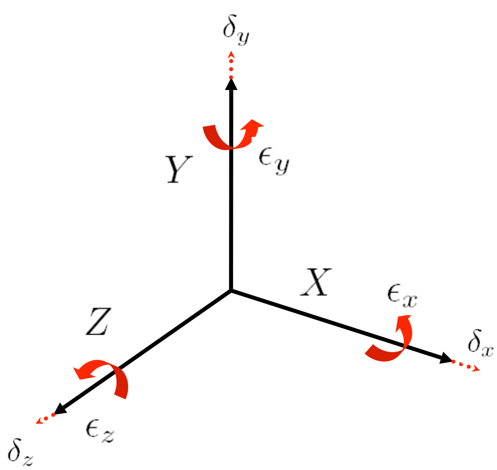

Figure 1: Coordinate system of component A with positional errors $\delta_x, \delta_y, \delta_z$ and rotational errors $\epsilon_x, \epsilon_y, \epsilon_z$

Lets first look at the generalized form of a HTM:

$$T = \begin{bmatrix} O_{ix} & O_{iy} & O_{iz} & d_x \\ O_{jx} & O_{jy} & O_{jz} & d_y \\ O_{kx} & O_{ky} & O_{kz} & d_z \\ 0 & 0 & 0 & d_s \end{bmatrix} \tag{1}$$

where, O_{ix} etc., are direction cosines representing the orientation of the body to the reference frame, d_x, d_y, d_z are the displacements in the principal directions, and d_s is scaling relative to the origin.

For the case of a system with no error, the HTM is:

$$T_{ideal} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

For a system with positions errors $\delta_x, \delta_y, \delta_z$ and rotational errors $\epsilon_x, \epsilon_y, \epsilon_z$, the HTM relative to the coordinate system R (^RT_A) can be constructed as follows. In this system the rotational terms are defined as:

- Rotation in **x** is **roll**
- Rotation in **y** is **yaw**
- Rotation in **z** is **pitch**

A **roll** of ϵ_x changes the following terms in the idealized HTM:

$$\begin{gathered}\sin\epsilon_x \text{ in } +Z \text{ direction for } Y \Rightarrow O_{ky} = \sin\epsilon_x \\ \cos\epsilon_x \text{ in } +Y \text{ direction for } Y \Rightarrow O_{jy} = \cos\epsilon_x \\ -\sin\epsilon_x \text{ in } +Y \text{ direction for } Z \Rightarrow O_{jz} = -\sin\epsilon_x \\ \cos\epsilon_x \text{ in } +Z \text{ direction for } Z \Rightarrow O_{kz} = \cos\epsilon_x\end{gathered}$$

Hence, we have:

$$T_{roll} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\epsilon_x & -\sin\epsilon_x & 0 \\ 0 & \sin\epsilon_x & \cos\epsilon_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

A **yaw** of ϵ_y changes the following terms in the idealized HTM:

$$\begin{gathered}-\sin\epsilon_y \text{ in } +Z \text{ direction for } X \Rightarrow O_{kx} = -\sin\epsilon_y \\ \cos\epsilon_y \text{ in } +X \text{ direction for } X \Rightarrow O_{ix} = \cos\epsilon_y \\ \sin\epsilon_y \text{ in } +X \text{ direction for } Z \Rightarrow O_{iz} = \sin\epsilon_y \\ \cos\epsilon_y \text{ in } +Z \text{ direction for } Z \Rightarrow O_{kz} = \cos\epsilon_y\end{gathered}$$

Hence, we have:

$$T_{yaw} = \begin{bmatrix} \cos\epsilon_y & 0 & \sin\epsilon_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\epsilon_y & 0 & \cos\epsilon_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

A **pitch** of ϵ_z changes the following terms in the idealized HTM:

$$\begin{gathered}\sin\epsilon_z \text{ in } +Y \text{ direction for } X \Rightarrow O_{jx} = \sin\epsilon_z \\ \cos\epsilon_z \text{ in } +X \text{ direction for } X \Rightarrow O_{ix} = \cos\epsilon_z \\ -\sin\epsilon_z \text{ in } +X \text{ direction for } Y \Rightarrow O_{iy} = -\sin\epsilon_z \\ \cos\epsilon_z \text{ in } +Y \text{ direction for } Y \Rightarrow O_{jy} = \cos\epsilon_z\end{gathered}$$

Hence, we have:

$$T_{pitch} = \begin{bmatrix} \cos\epsilon_z & -\sin\epsilon_z & 0 & 0 \\ \sin\epsilon_z & \cos\epsilon_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

For a system with roll, pitch, yaw errors, the resultant HTM can be calculated as $T_{roll} * T_{pitch} * T_{yaw}$ and is:

$$T_{roll,pitch,yaw} = \begin{bmatrix} \cos\epsilon_z \cos\epsilon_y & -\sin\epsilon_z & \cos\epsilon_z \sin\epsilon_y & 0 \\ \cos\epsilon_x \sin\epsilon_z \cos\epsilon_y + \sin\epsilon_x \sin\epsilon_y & \cos\epsilon_x \cos\epsilon_z & \cos\epsilon_x \sin\epsilon_z \sin\epsilon_y - \sin\epsilon_x \cos\epsilon_y & 0 \\ \sin\epsilon_x \sin\epsilon_z \cos\epsilon_y - \cos\epsilon_x \sin\epsilon_y & \sin\epsilon_x \cos\epsilon_z & \sin\epsilon_x \sin\epsilon_z \sin\epsilon_y + \cos\epsilon_x \cos\epsilon_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

We can simplify the above HTM with the following two assumptions [1]:

- Since $\epsilon_{x,y,z} << 1$, $\sin\epsilon_{x,y,z} \approx \epsilon_{x,y,z}$ and $\cos\epsilon_{x,y,z} \approx 1$
- Higher order error terms (example: $\sin\epsilon_x \sin\epsilon_y$) can be approximated to 0

Applying these assumptions, the HTM becomes:

$$T_{roll,pitch,yaw} = \begin{bmatrix} 1 & -\epsilon_z & \epsilon_y & 0 \\ \epsilon_z & 1 & -\epsilon_x & 0 \\ -\epsilon_y & \epsilon_x & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

For a system with **displacement errors** $\delta_x, \delta_y, \delta_z$, the HTM is:

$$T_{displacement} = \begin{bmatrix} 1 & 0 & 0 & \delta_x \\ 0 & 1 & 0 & \delta_y \\ 0 & 0 & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

Combining rotational and displacement errors (again by multiplication and ignoring the higher order terms), we can formulate the (approximate) HTM for system A relative to the reference R as:

$$^{R}T_A = \begin{bmatrix} 1 & -\epsilon_z & \epsilon_y & \delta_x \\ \epsilon_z & 1 & -\epsilon_x & \delta_y \\ -\epsilon_y & \epsilon_x & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

If this system also had the deterministic displacement of x, y, z in each of the principal directions, the HTM becomes:

$$^{R}T_A = \begin{bmatrix} 1 & -\epsilon_z & \epsilon_y & x + \delta_x \\ \epsilon_z & 1 & -\epsilon_x & y + \delta_y \\ -\epsilon_y & \epsilon_x & 1 & z + \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

[1]In the case of HTMs for very precise machine tools and systems (such as the LODTM) this assumption is not valid.

2 Structural Errors

In this example we calculate the error gain matrix in locating a point on a machine tool structure (please see Figure 2). The machine tool structure consists of a Z slideway mounted on an X slideway, which is mounted on the ground (which is the reference). We are locating a point on top of the Z slideway of the structure. We are given the errors present in each structural element of the machine tool (these errors can come from experimental measurements or from the manufacturer's specifications).

In this example we use the approximate HTM formulation from Equation 9.

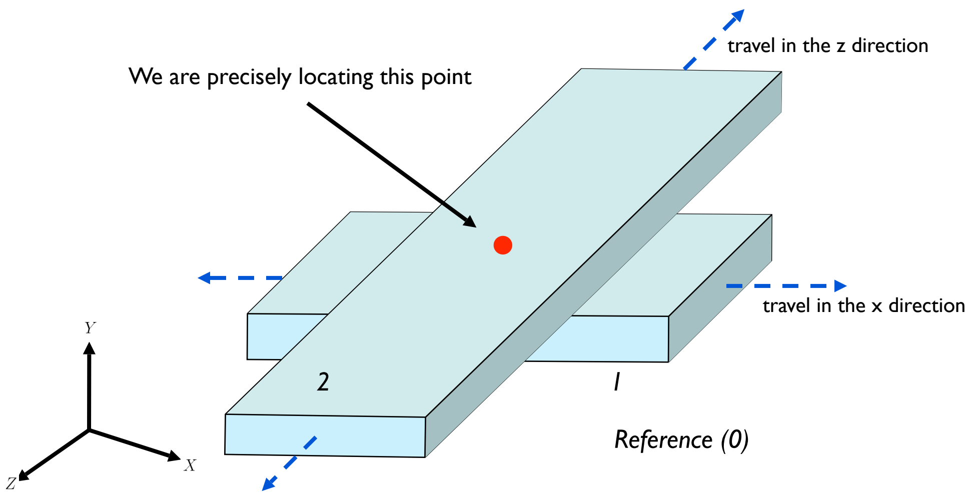

Figure 2: Machine tool structure

The structural loop for the machine tool is shown in Figure 3.

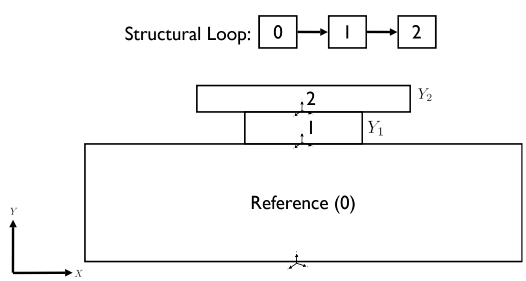

Figure 3: Structural loop

The machine tool has the following errors in Component 1.

- Finite positional error in x: δ_x
- Finite yaw error: ϵ_{y1}
- Finite pitch error: ϵ_{z1}

Component 1 is moving and has a variable displacement x in the X direction, and a fixed displacement Y_1 in the Y direction.

Hence, the HTM of Component 1 w.r.t the reference is as follows:

$$^{0}T_1 = \begin{bmatrix} 1 & -\epsilon_{z1} & \epsilon_{y1} & x+\delta_x \\ \epsilon_{z1} & 1 & 0 & Y_1 \\ -\epsilon_{y1} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

The machine tool has the following errors in Component 2.

- Finite positional error in z: δ_z
- Finite yaw error: ϵ_{y2}

Component 2 is moving and has a variable displacement z in the Z direction, and a fixed displacement Y_2 in the Y direction.

Hence, the HTM of Component 2 w.r.t Component 1 is as follows:

$$^{1}T_2 = \begin{bmatrix} 1 & 0 & \epsilon_{y2} & 0 \\ 0 & 1 & 0 & Y_2 \\ -\epsilon_{y2} & 0 & 1 & z+\delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

Consider that the point on the machine tool needs to be located at position (t_x, t_y, t_z) (relative to the Component 2). This position, relative to the reference is:

$$P_{ideal} = \begin{bmatrix} t_x + x \\ t_y + Y_1 + Y_2 \\ t_z + z \\ 1 \end{bmatrix} \tag{13}$$

While actual position is:

$$P_{actual} =^{0} T_1 *^{1} T_2 * \begin{bmatrix} t_x \\ t_y \\ t_z \\ 1 \end{bmatrix} \tag{14}$$

Hence the error in the point is:

$$P_{Error} = P_{actual} - P_{ideal} \tag{15}$$

and is calculated as:

$$P_{Error} =^{0} T_1 *^{1} T_2 * \begin{bmatrix} t_x \\ t_y \\ t_z \\ 1 \end{bmatrix} - \begin{bmatrix} t_x + x \\ t_y + Y_1 + Y_2 \\ t_z + z \\ 1 \end{bmatrix} \tag{16}$$

$$\Rightarrow \begin{bmatrix} \delta_{xT} \\ \delta_{yT} \\ \delta_{zT} \\ 1 \end{bmatrix} = \begin{bmatrix} (1-\epsilon_{y1}\epsilon_{y2})\, t_x - \epsilon_{z1} t_y + (-\epsilon_{y2}+\epsilon_{y1})\, t_z - \epsilon_{z1} Y_2 + \epsilon_{y1}(z+\delta_z) + \delta_x - t_x \\ \epsilon_{z1} t_x - \epsilon_{z1}\epsilon_{y2} t_z \\ (-\epsilon_{y1}-\epsilon_{y2})\, t_x + (\epsilon_{y1}\epsilon_{y2}+1)\, t_z + \delta_z - t_z \end{bmatrix} \tag{17}$$

From this formulation we can identify the error gain matrix. The error gain matrix identifies the "amplification" of the rotational errors in the positional error. The error matrix for the point position is:

$$\begin{array}{c|ccc} & \delta_{xT} & \delta_{yT} & \delta_{zT} \\ \hline \epsilon_{y1} & -\epsilon_{y2}t_x + t_z + z + \delta_z & 0 & -t_x + \epsilon_{y2}t_z \\ \epsilon_{z1} & -t_y - Y_2 & t_x - \epsilon_{y2}t_z & 0 \\ \epsilon_{y2} & -\epsilon_{y1}t_x - t_z & \epsilon_{z1}t_z & -t_x + \epsilon_{y1}t_z \end{array} \tag{18}$$

ME220 - PRECISION MANUFACTURING

Homework 4: Error Budgets

Joshua Chien, Moneer Helu, & Clinton Warren

10/30/2008

1 Selected Machine Tool & its Structural Loop

We selected a Mori Seiki NV1500DCG for our analysis. As Figure 1(b) shows, the structural loop of the machine tool follows the standard 'C' shape we expect. The ground is included as part of our model of the Mori Seiki machine tool because the spindle is assumed to be essentially disconnected from the parts of the machine tool that holds the workpiece.

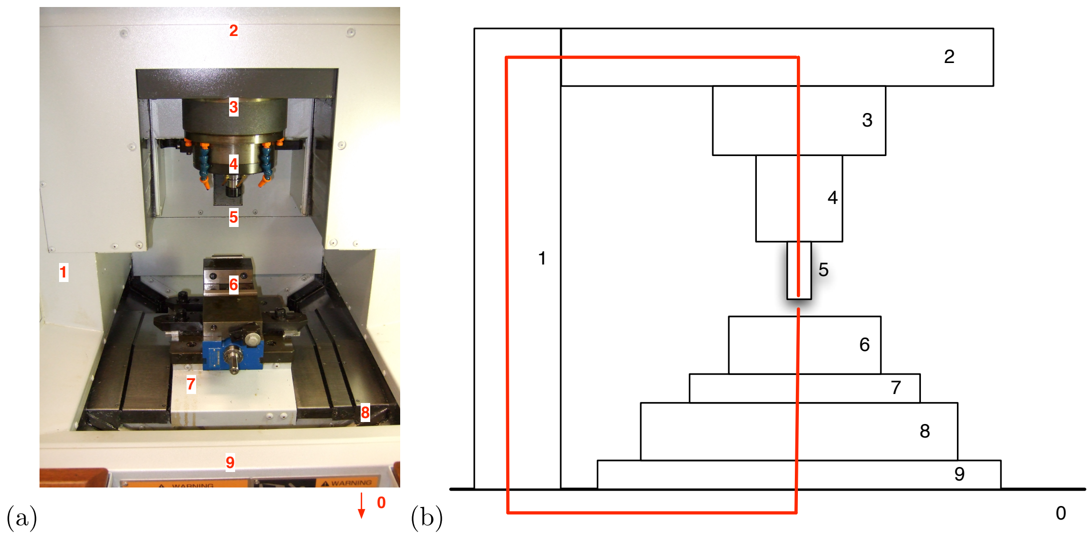

Figure 1: Components of Mori Seiki machine tool (a) indicated on image of tool workspace (corresponding names of numbered components can be found in Section 2) & (b) schematically represented with structural loop indicated

2 Machine Components, Axes, & Offsets

We decided to divide the Mori Seiki machine tool into 9 components (see Figure 1(a)): (1) vertical support, (2) horizontal support #1, (3) spindle head, (4) spindle, (5) tool, (6) workpiece holder, (7) x-stage, (8) y-stage, (9) horizontal support #2. We decided to lump the many different parts of the Mori Seiki machine tool into the 10 components given based on the scope of our analysis. For example, we are not interested in the minute errors at the interface of any two structural parts and so we decided to lump all structural parts of the machine tool into a single support element. Or, we are not interested in the small errors that occur when gears mate and so we decided to also lump the motor, gears, and structural elements of the spindle head into one component. Furthermore, we have included two separate vertical supports because of our original assumption that the spindle and workpiece holder are essentially disconnected in the machine tool.

We selected the location of the reference and component axes in order to simplify our homogeneous transfer matrices (HTMs). For all components except component #1 (vertical support), individual component axes are placed on the axis running through the center of the tool and spindle. The axis for the ground reference is also placed on the axis running through the center of the tool and spindle. The component axis for component #1 (vertical support) was placed near the top of the vertical support in line with the component axis for component #2 (horizontal support #1); this was done in order to simplify the HTM for the 1-2 interface. Figure 2 gives a graphical representation of the location of each axis. The offsets between each set of axes is given in Table 1.

3 Ideal HTMs

Using Table 1, we can define the HTM for each component of the machine tool assuming ideal behavior

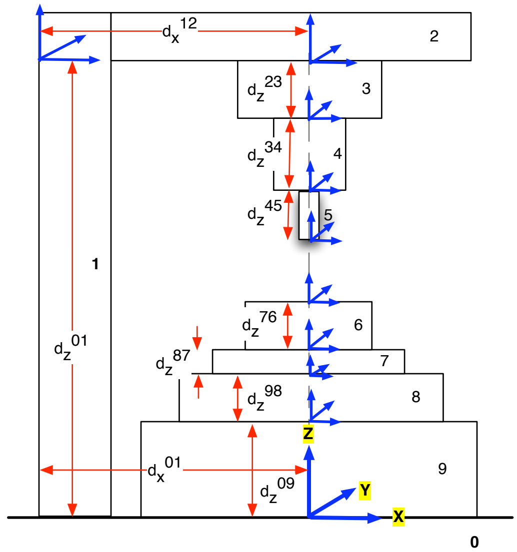

Figure 2: Location of reference and individual component axes in the Mori Seiki machine tool

Table 1: Offsets between each set of axes in the structural loop of the Mori Seiki machine tool

Component A	Component B	x-offset	y-offset	z-offset
0	1	-d_x^{01}	0	d_z^{01}
1	2	d_x^{12}	0	0
2	3	0	0	-d_z^{23}
3	4	0	0	-d_z^{34}
4	5	0	0	-d_z^{45}
0	9	0	0	d_z^{09}
9	8	0	0	d_z^{98}
8	7	0	0	d_z^{87}
7	6	0	0	d_z^{76}

of the component as follows:

$$
{}^0T_1 = \begin{bmatrix} 1 & 0 & 0 & -d_x^{01} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_z^{01} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad
{}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & d_x^{12} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad
{}^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_z^{23} \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^3T_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_z^{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad
{}^4T_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_z^{45} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad
{}^0T_9 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_z^{09} \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^9T_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_z^{98} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad
{}^8T_7 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_z^{87} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad
{}^7T_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_z^{76} \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

where ${}^aT_b$ is the HTM from component a to component b, and d_i^{ab} is the offset in the i-direction between axes of component a and component b.

4 Errors in Each Component

Of course, each component will not have ideal behavior. Theoretically, each component could have finite errors in all six degrees of freedom. However, using some reasonable engineering assumptions, we can identify what are likely the most significant errors in our tool. In all cases, errors are assumed to exist in the intended direction of actuation in a component due to imperfect actuation, encoding, and sensing. All components are assumed stiff to axial compression and tension, but not necessarily to bending.

- ***Component #1 - Vertical Support:*** No errors
  The vertical support component of the outside frame is assumed to be perfectly rigid given its large size and mass compared to other components. Cantilever effects are highly mitigated due to the presence of vertical supports on both sides of the tool, which creates an 'O' shape rather than the 'C' shape assumed in most analyses.

- ***Component #2 - Horizontal Support #1:*** No errors
  Using a similar argument as in component #1, the horizontal support beam is assumed to be rigidly fixed and perfectly stiff.

- ***Component #3 - Spindle Head:*** Errors in z translation (δ_z^3) & y rotation (ϵ_y^3)
  The spindle head is designed to translate in the vertical z-direction in relation to component #2, the horizontal support. Due to errors in the encoder and imperfect actuation, we can expect significant errors in z-displacement. Also, friction, slop, and other uneven effects on the sides of this component will lead to rotational errors about the y-axis during actuation.

- ***Component #4 - Spindle:*** Errors in x translation (δ_x^4) & y translation (δ_y^4)
  Run-out and wobble during rotation will lead to errors in both x and y translation at the end of the spindle. Due to the geometry of the fixturing, rotations about the x and y axis are considered to be negligible. In addition, because the spindle is designed to rotate about the z direction relative to component #3, we can expect errors in the z-rotation of component #4. However, this error is not positional in nature, and so we can neglect it in our analysis.

- ***Component #5 - Tool:*** Errors in x translation (δ_x^5) & y translation (δ_y^5)
  The tool is assumed to be rigidly fixed to the spindle and therefore exhibits no significant errors in rotation. In addition, translational errors in the vertical direction are considered negligible due to the stiffness of the tool and the lack of a moment arm in that direction. On the other hand, x and y translational errors are considered significant in this analysis due to deflection at the tip of tool which are caused by cutting forces.

- ***Component #6 - Workpiece Holder:*** No errors
  The workpiece holder is considered completely rigid and perfectly fixed to component #7.

- ***Component #7 - x-stage:*** Errors in x translation (δ_x^7) & z rotation (ϵ_z^7)
  Since the x-stage is actuated in the x direction, we expect significant errors in x translation. In addition, rotational errors about the z axis are expected due to friction, play, and unbalanced actuation at the edges of the stage. Since the x-stage is much wider than it is thick, rotational errors about the y-axis are less significant and can be neglected. Translation in the z and y directions is negligible due to the method with which the x-stage is fixed to the y-stage.

- ***Component #8 - y-stage:*** Errors in y translation (δ_y^8) & z rotation (ϵ_z^8)
  Our argument for errors in the y-stage follows the same logic as that in component #7.

- ***Component #9 - Horizontal Support #2:*** No errors
  This component is assumed to be perfectly rigid and is attached to the ground with no freedom of movement.

5 Reformulation of HTMs with Error Terms

Taking the errors defined in Section 4 into account, we must re-define our component HTMs as follows:

$$
{}^0T_1 = \begin{bmatrix} 1 & 0 & 0 & -d_x^{01} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_z^{01} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & d_x^{12} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad {}^2T_3 = \begin{bmatrix} 1 & 0 & \epsilon_y^3 & 0 \\ 0 & 1 & 0 & 0 \\ -\epsilon_y^3 & 0 & 1 & \delta_z^3 - d_z^{23} \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^3T_4 = \begin{bmatrix} 1 & -\epsilon_z^4 & 0 & \delta_x^4 \\ \epsilon_z^4 & 1 & 0 & \delta_y^4 \\ 0 & 0 & 1 & -d_z^{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^4T_5 = \begin{bmatrix} 1 & 0 & 0 & \delta_x^5 \\ 0 & 1 & 0 & \delta_y^5 \\ 0 & 0 & 1 & -d_z^{45} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad {}^0T_9 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_z^{09} \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^9T_8 = \begin{bmatrix} 1 & -\epsilon_z^8 & 0 & 0 \\ \epsilon_z^8 & 1 & 0 & \delta_y^8 \\ 0 & 0 & 1 & d_z^{98} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^8T_7 = \begin{bmatrix} 1 & -\epsilon_z^7 & 0 & \delta_x^7 \\ \epsilon_z^7 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_z^{87} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad {}^7T_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_z^{76} \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

where δ_i^a is a translational error and ϵ_i^a is a rotational error in component a in the i-direction.

6 Error Gain Matrix

Using our simplified HTMs from Section 5, we can define an HTM that refers component #5 (tool) back to the reference axis on the ground (component #0) as well as another HTM that refers component #6 (the workpiece) back to the reference axis on the ground (component #0). The resulting HTMs can then be used to find the HTM of interest that relates component #5 (tool) to component #6 (workpiece)

$$
\begin{aligned}
{}^0T_5 &= {}^0T_1 *^1 T_2 *^2 T_3 *^3 T_4 *^4 T_5, \\
{}^0T_6 &= {}^0T_9 *^9 T_8 *^8 T_7 *^7 T_6, \\
\therefore^5 T_6 &= {}^0T_6 * \left({}^0T_5\right)^{-1}.
\end{aligned}
$$

If we assume that both rotational and translational errors in the Mori Seiki are very small, then we can approximate any higher order error terms to be negligible. This allows us to simplify ${}^5T_6$:

$$
{}^5T_6 = \begin{bmatrix} 1 & -\epsilon_z^7 - \epsilon_z^8 & -\epsilon_y^3 & \epsilon_y^3\left(d_z^{01} - d_z^{23}\right) - \delta_x^5 - \delta_x^4 + \delta_x^7 - d_x^{12} + d_x^{01} \\ \epsilon_z^7 + \epsilon_z^8 & 1 & 0 & -\left(\epsilon_z^7 + \epsilon_z^8 - \epsilon_z^4\right)\left(d_x^{12} - d_x^{01}\right) - \delta_y^4 - \delta_y^5 + \delta_y^8 \\ \epsilon_y^3 & 0 & 1 & -\delta_z^3 + \epsilon_y^3\left(d_x^{01} - d_x^{12}\right) - d_z^{01} + d_z^{23} + d_z^{34} + d_z^{45} + d_z^{76} + d_z^{87} + d_z^{98} + d_z^{09} \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

Because of the way in which we have defined our offsets, we can write:

$$
\begin{aligned}
d_x^{01} &= d_x^{12}, \\
d_z^{01} &= d_z^{09} + d_z^{98} + d_z^{87} + d_z^{76} + d_z^{45} + d_z^{34} + d_z^{23}.
\end{aligned}
$$

These relationships allow us to further simply ${}^5T_6$ to its final form:

$$
{}^5T_6 = \begin{bmatrix} 1 & -\epsilon_z^7 - \epsilon_z^8 & -\epsilon_y^3 & \epsilon_y^3\left(d_z^{01} - d_z^{23}\right) - \delta_x^5 - \delta_x^4 + \delta_x^7 \\ \epsilon_z^7 + \epsilon_z^8 & 1 & 0 & -\delta_y^4 - \delta_y^5 + \delta_y^8 \\ \epsilon_y^3 & 0 & 1 & -\delta_z^3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

Using the HTM ${}^5T_6$, we can define the error gain matrix. Even though error gain matrices are meant to identify the amplification of rotational errors (ϵ_i^a) in the positional error, we include the positional errors for completeness. As expected, the contribution of positional errors is ± 1 since these errors are not amplified when considering the overall position error.

	δ_{xT}	δ_{yT}	δ_{zT}
ϵ_y^3	$d_z^{01} - d_z^{23}$	0	0
ϵ_z^7	0	0	0
ϵ_z^8	0	0	0
δ_z^3	0	0	-1
δ_x^4	-1	0	0
δ_y^4	0	-1	0
δ_x^5	-1	0	0
δ_y^5	0	-1	0
δ_x^7	1	0	0
δ_y^8	0	1	0

7 Estimation of Error Terms in Mori Seiki

We selected the z-axis of the machine tool to numerically estimate the error terms. As the error gain matrix in Section 6 shows, the error in the z-axis of the machine tool, δ_{zT}, is composed of only one error term, δ_z^3, or the z-translation error of component #3 (spindle head). We can estimate δ_z^3 by using the data presented in Schmitz, et al. (2008) and assuming a worst-case scenario. Thus, we estimate that $\delta_z^3 \sim 4\mu$m and so $\delta_{zT} = -1 * \delta_z^3 \sim -4\mu$m.

8 Estimation of Error in Manufactured Part

If we were to face mill a rectangular prism on the Mori Seiki to a height of 50μm, then the error discussed in Section 7 (error in the z-axis of the machine tool) would cause the top surface of the prism to be machined incorrectly depending on the nature of the error. So, if the error is deterministic, then the top surface of the prism would remain flat, but the finished height of the prism would be 46μm since we have calculated that $\delta_{zT} \sim -4\mu$m (see Figure 3(a)). However, if the error is probabilistic, then the top surface of the prism would likely not remain flat and the finished height of the prism would vary randomly between a higher and lower threshold value. As Figure 3(b) shows, we can simulate the effect of a probabilistic error on the finished surface by assuming that the worst-case error, $\delta_{zT} \sim -4\mu$m, represents the maximum variation from the nominal commanded height of 50μm.

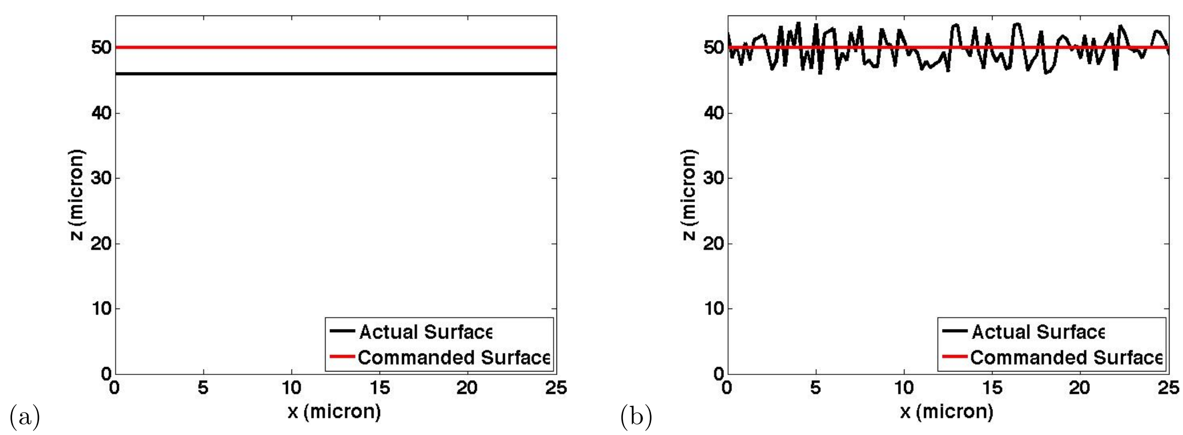

Figure 3: Comparison of the commanded surface to the surface actually machined for (a) deterministic errors, & (b) probabilistic errors

9 Error Compensation without Physical Re-design

There are several approaches that can be explored to compensate for errors in the z-axis of the machine tool. Perhaps the simplest approach is to first recognize that $\delta_{zT} \sim -4\mu$m. If we assume that this is a deterministic error, then we can account for it by using an offset of 4μm when planning the tool path. For example, the prism we explored in Section 8 could be properly machined if we commanded the machine to 54μm in stead of 50μm. However, the reality is that our error is very likely non-deterministic, and so adding an offset of 4μm may be over-compensating.

A better approach is to initially assume that the errors are non-deterministic and thus best solved via feedback control. Additional sensors capable of surviving the harsh conditions associated with machining (due to lubricants, chips, etc.) and detecting the tool location precisely would have to be properly integrated into the Mori Seiki such that they would not interfere with the machining process itself. Examples of such sensors would include optical encoders or interferometry. The feedback controller would have to be programmed and integrated into the existing machine tool controller. Although this approach may be more costly, an approach of this nature should enable a relatively good response capable of eliminating or at the very least significantly reducing errors in the z-axis of the machine tool. Ultimately, though, it is very difficult to successfully compensate for error that is itself already in the micron range. We can assume that the Mori Seiki machine is equipped with sophisticated encoders and controllers such that further improvements may be difficult to attain.

10 Error Compensation with Physical Re-design

Since the Mori Seiki is an expensive and precise machine tool, it is difficult to offer many easily implementable physical re-design suggestions. However, there are some basic principles that could be followed that may lead to improvements in the error compensation. With regard to the errors in the z-axis of the machine tool, we are interested in the z-translation error of component #3 (spindle head). Currently, this is a large part with a diameter of nearly a foot. One possible area for exploration would be to reduce the surface area which contacts the sliding mechanism of this component in order to reduce friction and errors caused by surface roughness and irregularities. Granted, we cannot reduce the surface area too much to ensure proper actuation of the spindle head, but we may be able to machine a smaller feature more precisely and uniformly if this change is implemented properly. In addition, different bearings could be considered, though it is unknown what type are currently used in the machine tool. Finally, a more accurate encoder could be integrated into this component. Analogue designs should also be considered and their benefits and costs assessed. Again, though, it is very difficult to successfully compensate for error on the micron scale. We can assume that the Mori Seiki machine is likely designed to minimize inherent errors, and so further improvements may be difficult to attain.

References

[1] Schmitz, T. L., Ziegert, J. C., Canning, J. S., Zapata, R. (2008). Case study: a comparison of error sources in high-speed milling. Precision Engineering, 32, pp. 126 - 133.